JHEP HYPER VERSION

RECEIVED: *August 30, 2001*, ACCEPTED: *October 22, 2001*
REVISED: *October 10, 2001*

The spectrum of the MSSM with non-standard supersymmetry breaking

James P.J. Hetherington

Cavendish Laboratory, University of Cambridge
Madingley Road, Cambridge, CB3 0HE, UK
E-mail: `jamesh@hep.phy.cam.ac.uk`

ABSTRACT: The Minimal Supersymmetric Standard Model (MSSM) can include two soft breaking terms which are often neglected: a non-analytic scalar trilinear coupling and a Higgsino bilinear term. A set of high-scale boundary conditions consistent with the reparameterisation invariance which the model possesses is obtained. The three-family renormalisation group equations for the MSSM with these terms are presented. The ranges of the universal high-scale values of these couplings which lead to an acceptable TeV-scale theory are obtained, as is the supersymmetric particle spectrum at this scale. The effect of the new terms on fine-tuning is presented. SOFTSUSY, an existing program for calculating SUSY particle spectra, has been used, with as few modifications as possible.

KEYWORDS: Supersymmetry Breaking, Beyond Standard Model, Supersymmetric Standard Model.

Contents

1. Introduction

A possibility for new physics beyond the standard model is supersymmetry (SUSY) [1, 2]. If fermionic generators are added to the bosonic generators of the Lorentz group, the new space-time symmetry is supersymmetry. In a theory with exact supersymmetry, all particles have a partner of equal mass but opposite spin-statistics. Cancellations between bosonic and fermionic loops prevent radiative corrections from driving scalar masses up to the highest scale present, assumed to be the GUT or Planck scale, 10^{16} to $10^{19}\,\mathrm{GeV}$, solving the naturalness problem of the Standard Model. In addition, the renormalised electromagnetic, weak, and strong couplings can be made to converge to an approximately common value at the grand unification scale.

Since supersymmetry is not observed amongst the already discovered particles it must be a broken symmetry. The superpartners of the observed particles must be undiscovered particles. The simplest possible SUSY extension of the Standard Model, with a new superpartner field for each standard model field, and the addition of a second Higgs scalar doublet, is called the minimal supersymmetric Standard Model, or MSSM.

In the MSSM, the supersymmetry breaking terms are often assumed to be flavour-independent and/or gauge-factor independent at the high scale, and to split as they evolve to low scales under the renormalisation group equations (RGEs). The model with this assumption is called the constrained MSSM (CMSSM) or minimal supergravity (mSUGRA). The universal nature of the SUSY breaking terms is motivated by the idea that supersymmetry breaking is mediated by some flavour-blind particle such as the graviton.

Within mSUGRA, the values of fundamental SUSY-breaking parameters are imposed as boundary conditions on the running SUSY-breaking masses and couplings at high scale, usually taken to be $M_X = 10^{16}\,\mathrm{GeV}$. Experimental masses of superparticles are obtained by evolving the lagrangian to the weak scale using the renormalisation group equations. A solution consistent with the mSUGRA boundary conditions at M_X and the known Standard Model constraints at M_Z must be found. This constitutes a boundary value differential equation problem, and can be solved numerically.

The MSSM lagrangian is usually claimed to include all possible "soft supersymmetry breaking" terms, terms which split the masses and couplings of particles and their superpartners, but which do not remove the supersymmetric protection against large radiative corrections to scalar masses. It is also supposed to include all possible SUSY conserving terms given the particle content.

In fact, the "R-parity violating terms", which, if unconstrained, allow proton decay and flavour changing neutral currents, are also possible, but conventionally excluded. If these terms are excluded, superparticles are created and destroyed only in pairs, giving rise to the usual SUSY phenomenology, where the lightest supersymmetric particle is stable.

There exists a further set of possible additional "non-standard" soft supersymmetry breaking (NSSB) terms allowed by the symmetries, as remarked upon in [1] and [3]–[7]: a non-analytic scalar trilinear coupling, and a Higgsino bilinear coupling:

$$\mathcal{L}^{\rm NSSB} = C^u_{ij} H_1^* \tilde{Q}_i \tilde{\bar{u}}_j + C^d_{ij} H_2^* \tilde{Q}_i \tilde{\bar{d}}_j + C^e_{ij} H_2^* \tilde{L}_i \tilde{\bar{e}}_j + \tilde{\mu} \tilde{H}_1 \tilde{H}_2 + \text{c.c.}\,, \tag{1.1}$$

where i, j are family indices, and weak isospin, spinor, and colour indices are suppressed. C^f_{ij} and $\tilde{\mu}$ are the new couplings.

In [3] the C terms are written down and then taken as zero. In [4] Feynman rules and mass matrices for these terms are presented. The non-standard terms are written down in [1], and including them in SUSY studies is advocated there. In [5] and [6] the NSSB terms are used to generate flavour mass hierarchies through radiative corrections.

In [7] 1-loop β-functions are obtained for a SUSY model with NSSB terms, and a general particle content and gauge group. These are then specialised to a one-family MSSM. The region of $m_0, \tan\beta$ parameter space where an acceptable electroweak vacuum results is obtained, for a given value of $\tilde{\mu}(M_X)$ and with $\mu = 0$. $C_0 = C^u/Y^u = C^d/Y^d = C^e/Y^e$ is allowed to vary in order to obtain an acceptable vacuum, as discussed in section 5 of this paper. When $C_0 = \tilde{\mu}$ the theory is supersymmetric (C and $\tilde{\mu}$ can be removed by a reparameterisation as discussed in section 4 of this paper.) This is a fixed point of the RGEs, and its stability is discussed in [7]. In [8] those authors present 2-loop NSSB β-functions for general particle content and gauge group, and further discuss the fixed-point properties of the NSSB RGEs.

In [9] the impact of NSSB terms on flavour-changing neutral currents in the MSSM is considered. In [10] the NSSB terms are included together with dimensionless higher-order scalar couplings to examine the origin of intermediate scale physics. In [11, 12] the NSSB terms are used in models of neutrino masses and rare processes in top decay. In [13] the NSSB terms are used to fix the correct mass of the b-quark in a model where the decay $b \to s\gamma$ is generated radiatively.

These previous works have not considered how the mSUGRA assumptions can be adapted to be used with NSSB, determined the size of the valid NSSB parameter space for fixed values of the standard MSSM parameters, presented mass-spectra for the NSSB MSSM, or calculated the fine-tuning of the model. These issues are examined in the present work.

In the following sections the MSSM with NSSB is considered in detail. Section 2 is a review of the reasons why the NSSB terms have previously been neglected. In section 3 numerical methods which can be used to simulate the renormalisation group equations are discussed. (For more detail on this question, consult appendix B.) In section 4 a reparameterisation invariance is introduced, which the inclusion of these terms gives to the MSSM. In section 5 we develop a version of the "constrained MSSM" or "minimal supergravity" (mSUGRA) high-scale boundary conditions consistent with the reparameterisation symmetry. In section 6 the calculation of the 3-family renormalisation group equations (RGEs) for this model is considered. These are presented in appendix A. The impact of these terms upon electroweak symmetry

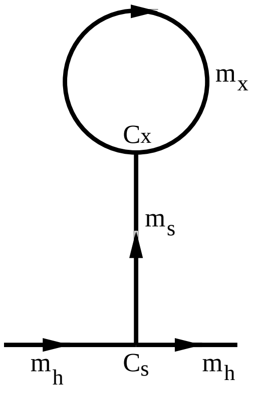

Figure 1: The scalar singlet tadpole correction to the Higgs propagator.

breaking is discussed in section 7. In section 8 an interesting subset of the particle masses for a standard mSUGRA point but with non-standard breakings is presented. In section 9 the reasons for the limits on the magnitudes of the non-standard couplings are explored. In section 10 numerical measures of the fine-tuning of the model are considered, and results for the fine-tuning presented and discussed. Appendix A lists the NSSB contribution to the 1-loop renormalisation group equations for the model. Appendix B examines the numerical methods used in this work.

2. Why have the NSSB terms traditionally been excluded?

The terms of eq. (1.1) have traditionally been neglected because they are thought to lead to large radiative corrections when there are scalar fields present which are uncharged under the entire gauge group.

This argument cannot be used in the context of the MSSM, which contains no singlets at the SUSY scale. In a model with some higher mass singlet field of mass m_s, together with a very massive field of mass m_X, the correction, (see figure 1) to the relation between the Higgs pole $m_{h,p}$ and running $m_{h,r}$ masses is:

$$m_{h,p}^2 = m_{h,r}^2(m_{h,p}) + C_s C_X S_4 \frac{m_X^2}{m_s^2} \ln\left(\frac{m_X^2}{m_{h,p}^2}\right) + \cdots , \tag{2.1}$$

where C_s is the coupling between the Higgs and the singlet, and C_X is the coupling between the singlet and the heavy field. S_4 is a collection of dimensionless constants. If the singlet field has a mass significantly below the high scale, the theory is thus unstable. However, if any singlet fields have masses of order M_X, such as in SUSY desert scenarios, then there is no problem with including NSSB terms.

3. Numerical calculation of SUSY particle spectra

In SUSY models, the supersymmetric lagrangian terms are fixed at the experimental scale M_Z from Standard Model particle data. The supersymmetry breaking terms are fixed at some high scale, M_X, according to a scheme such as mSUGRA.

The high- and low- scale boundary constraints, together with the renormalisation group equations, and the intermediate scale radiative electroweak symmetry breaking (REWSB) constraint (see section 7), form a system of differential equations; a "boundary value problem".

The numerical methods used to implement this procedure are discussed in detail in appendix B. Three such methods are discussed. The first is the formal "shooting method" where the constraint equation at one boundary is solved for those parameters which are unconstrained at the other boundary. This method cannot be used for SUSY, because of the complex form of the boundary conditions at the low scale.

The second method, which may be called the "drift method" involves alternate imposition of each set of boundary conditions. This constitutes a recurrence relation for successive values of those parameters unconstrained at one of the boundaries. The solution consistent with both sets of boundary conditions is a fixed point of the recurrence relation, but this point is not guaranteed to be stable or unique. If we assume the point is both stable and unique, however, simple repeated imposition of the boundary conditions, alternately at M_Z and M_X, will result in convergence toward the required solution. This is the method used in ISASUGRA [14], and in SOFTSUSY [15], the code which was modified for this work. The C++ inheritance structure of SOFTSUSY permitted large parts of the code of SOFTSUSY to be re-used.

Because the drift method becomes unstable for certain values of the NSSB parameters, a third approach has been developed, a "single-ended" shooting method, where the fixed point of the recurrence relation is solved for, using a Newton-Raphson method. This is further discussed in appendix B.

4. The NSSB lagrangian and μ-reparameterisation

The non-standard SUSY breaking terms to be added to the MSSM lagrangian are given in eq. (1.1). Using the usual formula [16] for extracting the full lagrangian from the superpotential, the following terms are those involving $m^2_{H_{1/2}}$, the Higgs soft masses, μ, the Higgs bilinear superpotential term, and the NSSB couplings:

$$\begin{aligned}\mathcal{L} = \cdots &+(\tilde{\mu}-\mu)H_1H_2 + \left(\mu^2+m^2_{H_1}\right)|H_1|^2 + \left(\mu^2+m^2_{H_2}\right)|H_2|^2 + \\ &+ \left(C^u_{ij}-Y^u_{ij}\mu\right)H_2^*\tilde{Q}_i\tilde{\bar{u}}_j + \left(C^d_{ij}-Y^d_{ij}\mu\right)H_2^*\tilde{Q}_i\tilde{\bar{d}}_j + \left(C^e_{ij}-Y^e_{ij}\mu\right)H_2^*\tilde{L}_i\tilde{\bar{e}}_j + \text{c.c.}\,,\end{aligned} \tag{4.1}$$

where $Y^{u/d/e}_{ij}$ are the Yukawa couplings.

Therefore, the lagrangian is invariant under the following reparameterisation:

$$\begin{aligned} \mu &\longrightarrow \mu + \delta \,, \\ \tilde{\mu} &\longrightarrow \tilde{\mu} + \delta \,, \\ m_H^2 &\longrightarrow m_H^2 - 2\mu\delta - \delta^2 \,, \\ C_{ij}^f &\longrightarrow C_{ij}^f + Y_{ij}^f \delta \,. \end{aligned} \tag{4.2}$$

This was partially pointed out in [7] where it was noted that one could write an MSSM lagrangian ($\tilde{\mu} = 0, C_{ij}^f = 0, \mu \neq 0$) in terms of an NSSB model with zero μ but non-zero C_{ij}^f and $\tilde{\mu}$. Here a different emphasis is used:, the lagrangian contains all terms allowed by the symmetries, and the reparameterisation is used like a gauge freedom, to allow calculations to be made in the most convenient "gauge".

The above lagrangian contains all terms involving μ, provided the Higgs bilinear breaking term is written as

$$\mathcal{L} = \cdots + m_3^2 H_1 H_2 \tag{4.3}$$

and not

$$\mathcal{L} = \cdots - B\mu H_1 H_2 \tag{4.4}$$

as is sometimes used, factoring out the μ. In the latter case, B would become a μ-reparameterisation dependent quantity. In order that this work can easily make contact with MSSM work conducted using B, we instead have the freedom to define B using the lagrangian term:

$$\mathcal{L} = \cdots - B(\mu - \tilde{\mu}) H_1 H_2 \,. \tag{4.5}$$

On the MSSM subspace of parameter space, where $\tilde{\mu}$ and C_{ij}^f are zero, this definition is consistent with eq. (4.4), but B remains μ-reparameterisation invariant.

5. The high-scale boundary conditions

As mentioned in section 3 the supersymmetric lagrangian terms are fixed at the experimental scale M_Z from Standard Model particle data. The supersymmetry breaking terms are fixed at some high scale, M_X, according to a scheme such as mSUGRA.

The exceptions to this are the superpotential μ-term, and the SUSY breaking term B. The magnitude of the Higgs vacuum expectation value (VEV) $v^2 = v_1^2 + v_2^2$, is known from the Z boson mass. In addition, a choice for $\tan\beta$ at M_Z is usually made. The values of μ and B fix both v and $v_1/v_2 = \tan\beta$, the ratio of the two Higgs vacuum expectation values, by minimisation of the Higgs effective potential (see section 7) which is a known function of the model parameters. Thus, the values of μ and B cannot be imposed as a boundary condition, but are allowed to vary to fit the Higgs VEVs.

While the above approach is the conventional one for MSSM calculations, in [7] the authors chose to use the reparameterisation freedom to take $\mu = 0$, solving the SUSY μ-problem. Instead of leaving μ and B at M_X unconstrained, so that they can vary to fit M_Z and $\tan\beta$, in this approach C and B are left unconstrained at M_X. (With C^e, C^d, C^u taken as equal at M_X and $\tilde{\mu}$ at M_X given as an extra boundary condition.)

In this investigation, we want to impose values for both C and $\tilde{\mu}$ as boundary conditions at M_X, with the same kind of universality assumptions as in mSUGRA, i.e. $C^f_{ij} = Y^f_{ij} C_0$. The parameter μ will remain in the model, and will, together with B, be varied to fit $\tan\beta$ and v. Since we do not choose $\mu = 0$ the model retains its reparameterisation symmetry.

This approach is perhaps preferable because it implements non-standard supersymmetry breaking with *minimal* changes to mSUGRA. However, there is a subtle difficulty, because we wish to use the mSUGRA assumption of universal high-scale soft scalar masses: the mass term for each of the sleptons, squarks and Higgs bosons should be the same at M_X. This assumption is inconsistent with the reparameterisation invariance. A reparameterisation changes the Higgs soft masses but not the squark and slepton masses, breaking the equality. To make the boundary condition well defined, we must therefore select the reparameterisation in which we want the scalars to be universal.

As will be seen in section 6, the renormalisation group running will take place in $\mu = 0$ reparameterisation. If we simply impose the mSUGRA boundary conditions in that reparameterisation, including a value for $\tilde{\mu}$, we cannot have μ as an unconstrained parameter at high scale.

Consider a model with m_0, $M_{1/2}$, C_0 and $\tilde{\mu}$ at M_X given as constraints, and values for μ (μ_S) and B (B_S) and for the other parameters unconstrained at high scale, but which are consistent with the low energy constraints. (This discussion is illustrated in table 1). On changing to the $\mu = 0$ reparameterisation used to run the RGEs, or the $\tilde{\mu} = 0$ reparameterisation used for some parts of the calculation, the values of μ and $\tilde{\mu}$ are mixed up, and only $(\tilde{\mu} - \mu)$ is known. In addition, the Higgs soft mass-squareds become different from those of the squarks and sleptons.

If we then run the RGEs, do the rest of one cycle of the calculation, and then return to M_X, we do not know how to separate μ and $\tilde{\mu}$. It seems we cannot return to the reparameterisation used to impose the constraints. However, the difference between the Higgs scalar masses and the squark and slepton masses must be that caused by the reparameterisation. Thus, the reparameterisation which makes the squark/slepton soft masses equal to the Higgs soft masses must be that which returns us to the initial state. This can be called the “universality reparameterisation”. Re-imposition of the high-scale boundary conditions then leaves the model unchanged.

$\overline{m_{\tilde{q}}}$	$\overline{m_h^2}$	$\tilde{\mu}$	μ	C^f	Action
MSSM as MSSM					
m_0	m_0^2	N/A	μ_c	N/A	Arrive at high scale
m_0	m_0^2	N/A	μ_c	N/A	Impose b.c.: at convergence this does nothing.
True NSSB					
m_0	$m_0^2 + \mu_c^2$	$-\mu_c + \tilde{\mu}_c$	0	$(C_0 - \mu_c)Y^f$	Arrive at high scale in $\mu = 0$ reparameterisation
m_0	m_0^2	$\tilde{\mu}_c$	μ_c	$C_0 Y^f$	Reparameterise $\delta = \sqrt{\overline{m_h^2} - \overline{m_{\tilde{q}}^2}} = \mu_c$
m_0	m_0^2	$\tilde{\mu}_c$	μ_c	$C_0 Y^f$	Apply b.c.
m_0	$m_0^2 + \mu_c^2$	$-\mu_c + \tilde{\mu}_c$	0	$(C_0 - \mu_c)Y^f$	Reparameterise $\delta = -\mu_c$
MSSM as NSSB					
m_0	$m_0^2 + \mu_c^2$	$-\mu_c$	0	$-\mu_c Y^f$	Arrive at high scale in $\mu = 0$ reparameterisation
m_0	m_0^2	0	μ_c	0	Reparameterise $\delta = \sqrt{\overline{m_h^2} - \overline{m_{\tilde{q}}^2}} = \mu_c$
m_0	m_0^2	0	μ_c	0	Apply b.c.
m_0	$m_0^2 + \mu_c^2$	$-\mu_c$	0	$-\mu_c Y^f$	Reparameterise $\delta = -\mu_c$

Table 1: The high-scale boundary condition(b.c.) part of the iterative procedure for determining the model parameters consistent with the chosen input parameters. The state of the model after convergence is achieved is shown, i.e., once the model has become invariant under the iterative numerical procedure used. Thus, the imposition of the high-scale boundary conditions leaves the parameters unchanged, as shown in the table. $\overline{m_{\tilde{q}}^2}$ is the average squark mass-squared, $\overline{m_h^2}$ is the average Higgs mass-squared. X_c is used to denote a particular value of the parameter named X.

As the calculation converges toward a solution consistent with both sets of boundary conditions, we need an estimator for the difference between the Higgs and squark/slepton soft mass-squareds at M_X. Until convergence is reached, $m_{H_1} \neq m_{H_2}$, $m_{\tilde{u}} \neq m_{\tilde{d}}$ etc. The choice used here is the difference between the average Higgs mass-squared and the average squark mass-squared.

Unlike the $\mu = 0$ approach this approach allows us, for any point in the MSSM, to model the same point, by using the NSSB with $C_0 = \tilde{\mu} = 0$. The resulting solution is identical with that obtained using SOFTSUSY normally, and the high-scale value of μ which the model selects to fit $\tan\beta$ is the same as in the MSSM. It is possible to gradually turn on $\tilde{\mu}$ and C_0 and move away from any MSSM point. The MSSM is a $D-2$-dimensional subset of the D-dimensional NSSB parameter space.

6. NSSB β-functions

Reference [7] presents general one-loop beta functions for a general SUSY model with arbitrary particle content and gauge group, allowing NSSB terms. The two-loop results are presented in [8]. Reference [7] also gives the RGEs specialised to the NSSB MSSM with one family and with the Yukawa couplings factored out of the parameters. For example, the lagrangian "A" soft breaking term is written as $m_{10} Y^t H_2 Q \bar{t}$.

An alternative presentation of the beta functions, better suited to numerical implementation, is the 3-family matrix form for the MSSM without NSSB used in [16], with the Yukawa matrices not factored out of the soft breakings. This version of the beta-functions has been calculated.

A subtle question is the impact of the μ-reparameterisation on the beta functions. The results of [7] are in $\mu = 0$ reparameterisation. The MSSM results of [16] have non-zero μ. We do not wish to re-calculate all of the MSSM part of the beta functions for the $\mu = 0$ reparameterisation. We wish to use part of the results of [16], modified according to the general results of [7], which are for $\mu = 0$. By considering the μ-reparameterisation behaviour of each quantity, and representing the beta functions as pieces with different reparameterisation behaviours, and with dependence on each parameter, it is possible to obtain for the soft masses:

$$\beta_{m_i^2}^{\mathrm{NSSB}} = \beta_{m_i^2}^{\mathrm{MSSM}} + \beta_{m_i^2}^{C,\tilde{\mu}}\,, \tag{6.1}$$

where the left hand side of each equation is the NSSB beta function for $\mu = 0$, β^{MSSM} is the MSSM $\mu \neq 0$ beta-function *with μ set to zero*, and $\beta^{C,\tilde{\mu}}$ is the piece from the general result of [7] which has $\tilde{\mu}$ or C dependence. A similar equation can be obtained for β_B, but it is simpler just to calculate it afresh. The beta functions for C and $\tilde{\mu}$ can be obtained from the general expression of [7].

The renormalisation group equations are presented in appendix A.

7. Radiative electroweak symmetry breaking and other low-scale considerations

The constraints given by analytical minimisation of the Higgs effective potential are used to obtain μ and B from $\tan\beta$ and v. This is done at the scale (M_S) where radiative corrections to the renormalisation group improved scalar potential have the smallest scale dependence, see below.

The form of the tree-level radiative electroweak symmetry breaking (REWSB) constraint equation for μ:

$$\frac{1}{2} M_Z^2 = \frac{m_{H_1}^2 + \mu^2 - \left(m_{H_2}^2 + \mu^2\right) \tan^2\beta}{\tan^2\beta - 1} \tag{7.1}$$

suggests that the resulting value of μ will be correct in any reparameterisation, because of the way in which the reparameterisation keeps $m_H^2 + \mu^2$ constant. This equation finds the μ value consistent with given Higgs soft masses. However, in the $\mu = 0$ reparameterisation, numerical implementation of this equation fails, because the resulting μ^2 value close to convergence toward zero is sometimes negative.

The values of the model parameters obtained after imposing the REWSB constraints, if compared in the same reparameterisation, must be the same whatever reparameterisation we impose eq. (7.1) in. This is a consequence of the invariance of the lagrangian, and can be verified by considering the application of a reparameterisation to the tree-level result eq. (7.1). In order that the REWSB routine be convergent around a positive value of μ^2, we can thus choose to impose the REWSB constraint after reparameterising so that $\tilde{\mu} = 0$.

The constraint eq. (7.1) is obtained by minimisation of the effective potential. In our study, such a constraint equation is used to obtain μ, and a similar expression to obtain B. The one-loop corrections to the effective potential are included, with the leading top/stop contributions from the NSSB couplings, and the calculation is performed at the renormalisation scale where the scale dependence of the 1-loop corrections is smallest, approximately $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. This scale, M_S is approximately 760 GeV for the mSUGRA parameters used here, and is intermediate between M_Z and M_X. It is believed that the scale at which the 1-loop result has the slowest scale dependence, the contribution from higher loops is smallest. This is motivated by ref. [17], which shows that for the MSSM Higgs potential, the tree-level and 1-loop results are closest at this scale. However, this relationship has been shown not to work as well for other potentials, such as MSSM charge-and-color-breaking potentials, in ref. [18].

The physical mass matrices for those particles affected by $\tilde{\mu}$ and C in $\mu = 0$ reparameterisation are given in [7]. The unaffected mass matrices can be used as originally implemented in SOFTSUSY.

The NSSB contributions to the mass matrices given in [7] are the leading, tree-level, NSSB corrections to the relationship between the physical particle mass and the running model parameters. SOFTSUSY includes some MSSM loop corrections in the mass matrices, but such corrections involving the NSSB couplings have not been added.

8. Results on the particle spectrum

As an example of the superparticle spectrum in NSSB, figures 2 and 3 show some of the masses on lines through NSSB parameter space passing through mSUGRA standard point II (defined using the sign conventions of [19]):

$$\begin{aligned} &m_0 = 400\,\text{GeV}\,; \qquad M_{1/2} = 400\,\text{GeV}\,; \qquad \tan\beta = 10\,; \\ &A_0 = 0\,; \qquad \mu > 0\,. \end{aligned} \tag{8.1}$$

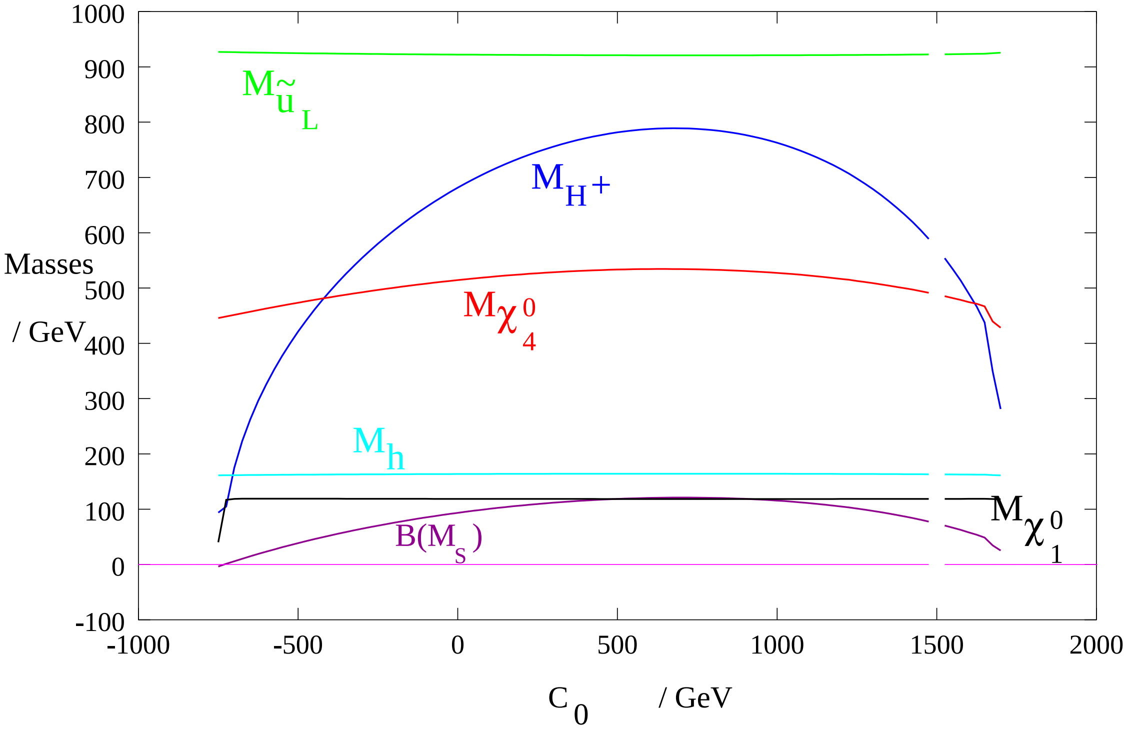

Figure 2: Some of the MSSM particle masses, and the B parameter at the REWSB scale, for non-zero values of the non-standard supersymmetry breaking parameter C_0. $C_0 = 0$ here is the mSUGRA point II: eq. (8.1). At the missing point, and the final point, the numerical method failed to converge correctly. Detail of this region can be seen in figure 7. The masses and parameters shown are those referred to in section 9, or those masses which vary significantly across the range. Those masses shown which do not change significantly have been chosen to indicate this fact.

The same point with the opposite sign of μ, has been investigated and shows very similar results, except with $C_0 \mapsto -C_0$ and $\tilde{\mu} \mapsto -\tilde{\mu}$.

Note that there is nothing special about the spectrum at the zero values of C_0 or $\tilde{\mu}$. The MSSM point already has broken supersymmetry. The NSSB terms break a few remaining equalities between SUSY breaking terms, but this has no sudden qualitative effect on the superparticle spectrum.

The masses and soft breakings for non-zero C_0 near mSUGRA point II show an extremum in figure 2. This occurs where the B parameter is equal at M_S and M_Z. This behaviour is a property of the RGEs for the system.

The very low masses of the normally heavier Higgs bosons $(A, H^{\pm}, H^0)$ for large negative values of C_0, as shown in figure 2, and the removal of all but the lightest neutralinos and charginos from the lower mass part of the spectrum (figure 3) for large negative values of $\tilde{\mu}$, might show interestingly different phenomenology. This may be worth investigating with ISAJET [14] or HERWIG [20].

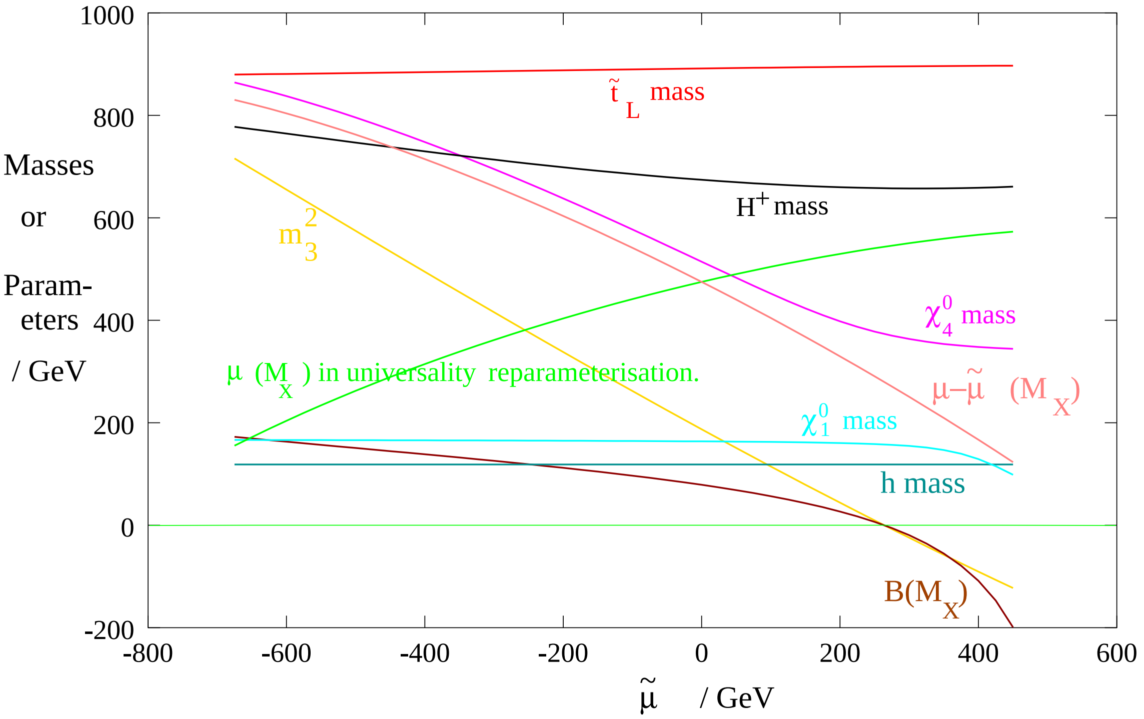

Figure 3: Some of the MSSM parameters and particle masses for non-zero values of the non-standard supersymmetry breaking parameter $\tilde{\mu}$. $\tilde{\mu} = 0$ here is the mSUGRA point II: eq. (8.1).

9. Parameter space constraints

What determines the largest positive and negative values of C_0 and $\tilde{\mu}$ allowed? For very large negative C_0 both the eigenvalues of the second differential matrix of the Higgs 1-loop effective potential at M_S become negative at $\langle H_1 \rangle = \langle H_2 \rangle = 0$. This is in contrast with the usual MSSM case where only one of the eigenvalues is negative — a phase transition has occurred. In this region, the REWSB constraint (eq. (7.1)) fails to correctly place the minimum of the Higgs effective potential at the required values of v and $\tan\beta$, placing a saddle point there instead. See figure 2, where the Higgs masses become small as we approach the transition at $C_0 \sim -750\,\mathrm{GeV}$. The shape of the surface below the transition is shown in figure 4. Note that eq. (7.1) assumes that the point found does correspond to the physical minimum, so we do *not* have a correct determination of the superspectrum in the unusual phase.

In order to obtain the above understanding, and obtain figure 4, code was written to obtain the full shape of the Higgs effective potential away from the minimum, including the dominant 1-loop corrections.

We should next attempt to resolve the question: is there a point in the parameter space where this new phase yields physical values of the particle masses? For the parameter set shown in figure 4 the Higgs VEV is much too large, re-

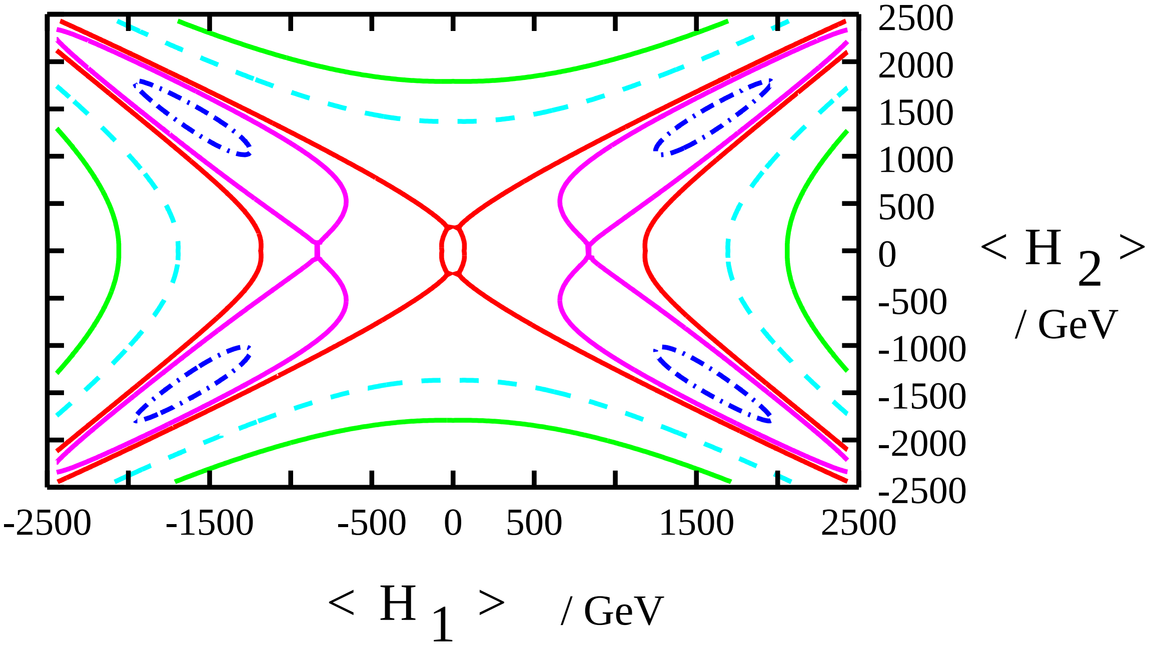

Figure 4: The Higgs effective potential surface for large values of C_0 near mSUGRA point II. (This plot was made with $C_0 = -800\,\mathrm{GeV}$.) Note the saddle point at $\tan\beta = 10, \langle v\rangle = 243\,\mathrm{GeV}$, i.e. at $\langle H_2\rangle \sim 240, \langle H_1\rangle \sim 0$. The central (red, solid) contour is $-1 \times 10^8\,\mathrm{GeV}^4$, the next contour (magenta, solid) is at $-8.5 \times 10^9\,\mathrm{GeV}^4$, the small elliptical contours (blue, dot-dashed) are at $-2 \times 10^{10}\,\mathrm{GeV}^4$, the blue, dashed contour is $10^{11}\,\mathrm{GeV}^4$, and the outermost (green, solid) contours are at $2 \times 10^{11}\,\mathrm{GeV}^4$.

sulting in a value of M_Z which is too large. Perhaps, for a given pair of $\tan\beta$ and $|\langle H\rangle|$, more than one set of values for the soft breakings is possible, corresponding to each of the two phases. Although, at tree level, eq. (7.1) suggests a unique value of the parameters for a given $\tan\beta$ and $|\langle H\rangle|$, it must actually be solved iteratively when the loop corrections are taken into account, because these corrections depend on the soft SUSY breakings. It is therefore possible that this iterative procedure possesses more than one solution. However, after an extensive search of parameter space, nowhere was the new phase found to be relevant to real physics.

There exist points in parameter space where the "universality reparameterisation" discussed in section 4 does not exist after imposing the low-energy constraints and running up to the high scale: the required value of μ^2 in the universality reparameterisation at high scale becomes negative. At such points the Higgs soft masses cannot, for positive μ^2 be made equal to the sfermion soft masses, while maintaining consistency with the electroweak symmetry breaking conditions. If this equality is forced, there is no positive value of μ^2 for which electroweak symmetry breaking occurs. This provides the limit on large negative values of $\tilde{\mu}$ at mSUGRA point II.

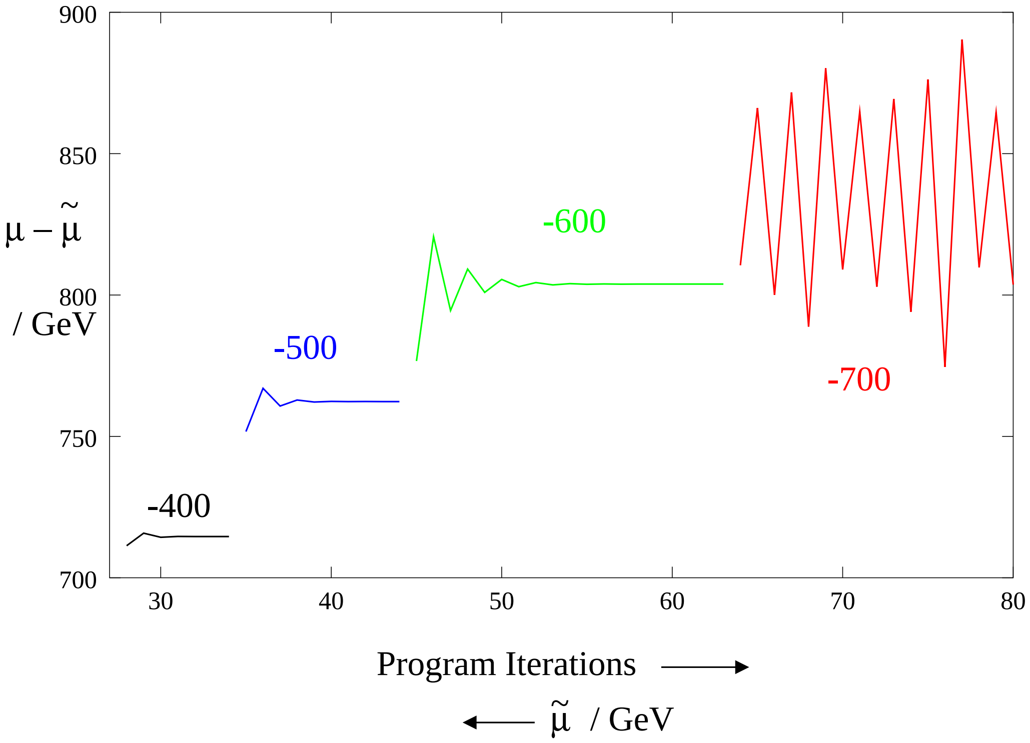

Figure 5: At mSUGRA point II, the recurrence relation solution to the SUSY RGEs converges more and more slowly for increasingly negative values of $\tilde{\mu}$. Each separate convergence in the plot is for a different value of $\tilde{\mu}$, with $\tilde{\mu}$ becoming more negative to the right. Note that for sufficiently large negative values of $\tilde{\mu}$, the method fails to converge at all. The numbers near the lines show the value of $\tilde{\mu}$ used for that point, in GeV.

It is for these large negative values of $\tilde{\mu}$ at mSUGRA point II that the recurrence relation approach becomes unstable. As shown in figure 5, the convergence toward the fixed point becomes steadily slower, before it finally becomes unstable. A naive belief in the recurrence relation approach would result in an underestimate of the size of the valid parameter space. The recurrence relation approach becomes unstable at $\tilde{\mu} \sim -675\,\text{GeV}$, while the single-ended drift method explained in appendix B extends the region which can be explored as far as $\tilde{\mu} \sim -775\,\text{GeV}$. Even that method cannot reach the physical limit, due to numerical instabilities caused by early iterations entering the unphysical region. This is illustrated in figure 6.

For large positive $\tilde{\mu}$ at mSUGRA point II, the bound on $\tilde{\mu}$ due to the value of B runs to extremely large negative values at high scale, making the calculation unstable. This is an artificial consequence of using B rather than m_3^2. As shown in figure 3, $m_3^2 = B(\mu - \tilde{\mu})$ is finite, but $\mu - \tilde{\mu}$ is becoming small, so B is singular. The calculation becomes unstable here due to the large value of B, but the upper bound on the value of $\tilde{\mu}$ comes from the lightest neutralino mass, which is becoming small as $(\mu - \tilde{\mu})$ does.

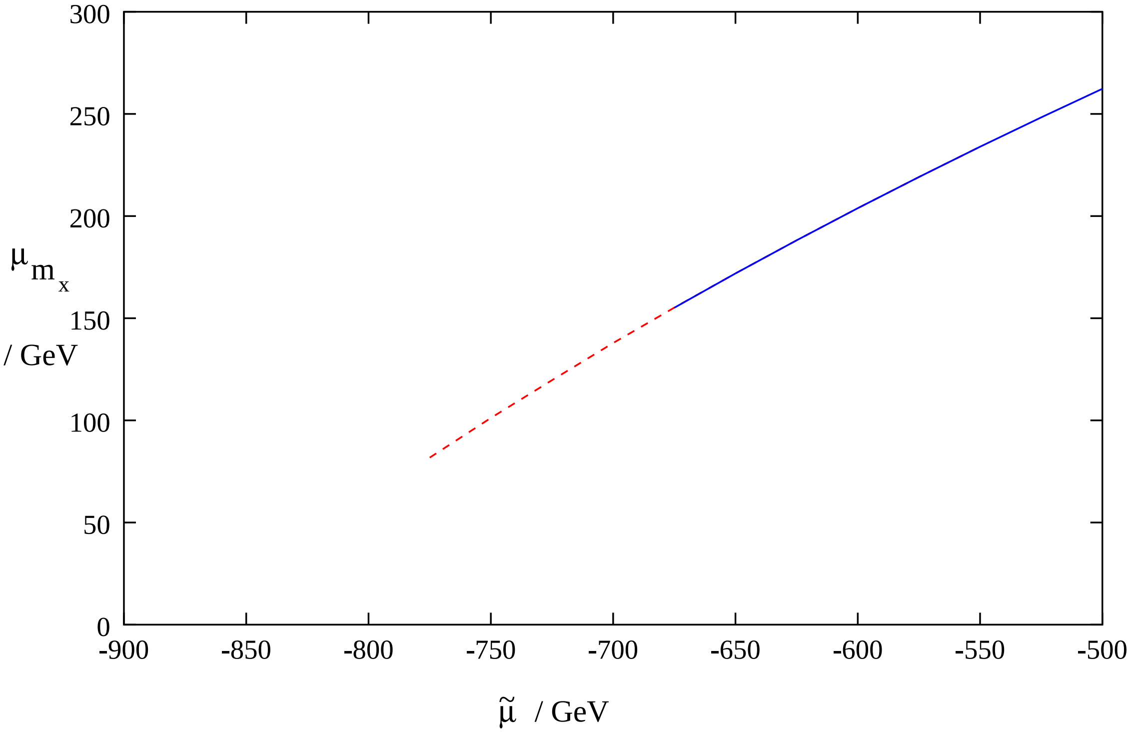

Figure 6: The value of μ in universality reparameterisation at M_X for non-zero $\tilde{\mu}$ near mSUGRA point II. The dotted part of the line shows the extra reach provided by the single-ended drift method.

For large positive values of C_0, above around $C_0 \sim 1500\,\text{GeV}$, the drift method becomes non convergent. The single-ended shooting method discussed in section 3 and appendix B takes us further, and is used to produce figure 2, which takes us up to $C_0 \sim 1675\,\text{GeV}$. Extrapolating, it appears that the physical limit occurs for the same reason as for large negative C_0, i.e. the Higgs masses become small. However, the physical limit cannot be reached even with the single-ended shooting method, for the reason discussed above. With care, some useful data can be teased out of this region (see figure 7), showing that the physical limit is around $C_0 \sim 1675\,\text{GeV}$.

10. Fine-tuning

Equation (7.1) shows that, with m_{H_1} and m_{H_2} of order the mass scale of the scalar superparticles, a cancellation is required so that M_Z can take its known value. The larger the typical mass of the scalar superparticles (controlled by the mSUGRA parameter m_0) the more finely tuned this calculation must be. One possible numerical measure of the delicacy of this calculation is [21, 22]:

$$c_a \equiv \left| \frac{\partial \ln M_Z^2}{\partial \ln |a|} \right| . \tag{10.1}$$

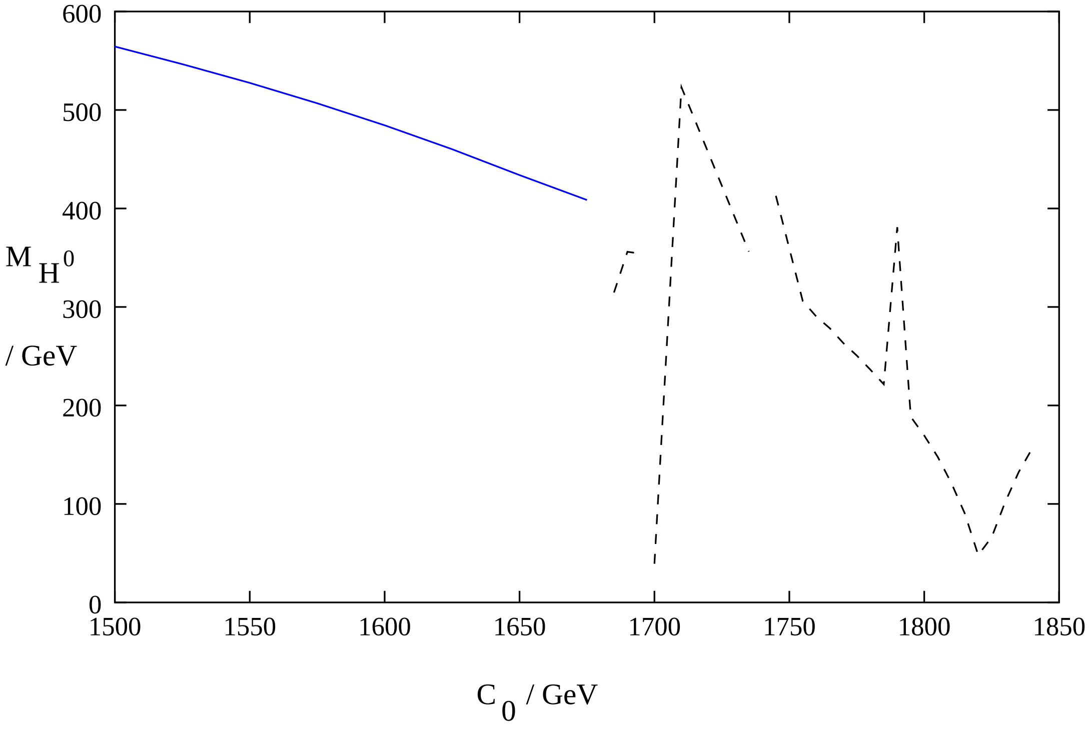

Figure 7: The mass of the heavy neutral Higgs particle, for non-zero C_0 near mSUGRA point II. All the data was produced using the single-ended shooting method. The dashed part of the line is the region near the physical limit where transients spoil the stability of the method. The figure shows the outcome of the method, whether or not it has formally converged.

The fine-tunings with respect to each model parameter a are calculated, and the overall fine-tuning of the model is defined to be $c = \max(c_a)$.

Here, there is a subtlety, again caused by the presence of the reparameterisation invariance: the same lagrangian may be described by a different choice of parameters. For example, one could choose, instead of C_0 and $\tilde{\mu}$, to reparameterise so that $C_0 = 0$, and use $(\overline{m_{\tilde{q},\tilde{l}}} - \overline{m_h})$ as the other parameter. Then, one would get different fine tuning values for the same lagrangian, depending on ones choice of parameter set. If this situation is thought to be unsatisfactory, the solution is to take the fine tuning with respect to the reparameterisation-invariant combinations of parameters appearing in the lagrangian, for example, $(\tilde{\mu} - \mu)$.

On the other hand, one may view it as acceptable that the fine-tuning depends upon the choice of reparameterisation, given that the fine-tuning measure is supposed to be a measure of ones satisfaction with the deeper model lying beyond the lagrangian being studied. A parameterisation choice reveals something of ones assumptions about the beyond-the-MSSM physics giving rise to the MSSM-type lagrangian.

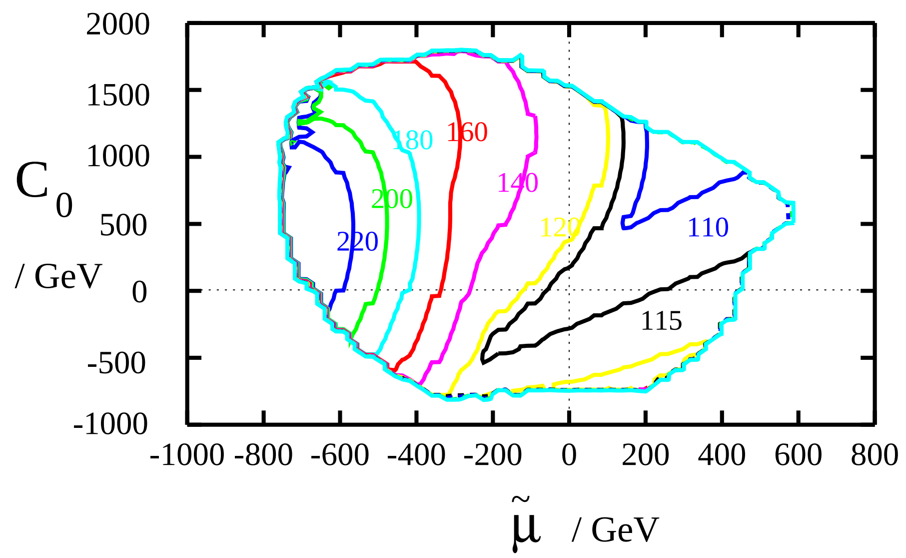

Figure 8: Fine tuning contours for non-standard supersymmetry breaking, with fine-tuning differentials taken with respect to invariant combinations of parameters. The plane is shown parameterised in terms of C_0 and $\tilde{\mu}$. The outermost line shows the edge of the valid parameter space, according to the drift method.

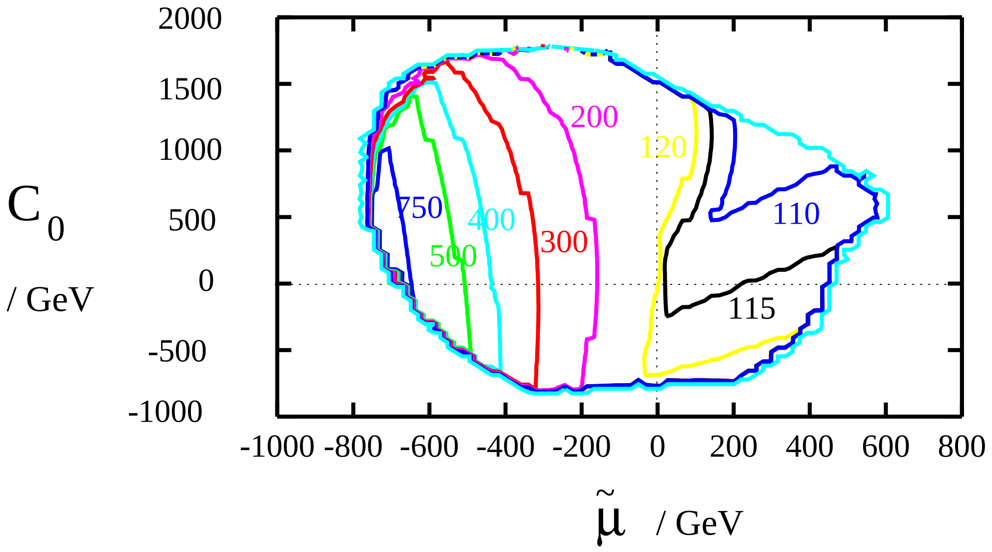

Figure 9: Fine tuning contours for non-standard supersymmetry breaking, with fine-tuning differentials taken with respect to the usual mSUGRA parameters, C_0 and $\tilde{\mu}$.

Figure 8 shows fine-tuning contours near mSUGRA point II with respect to invariant combinations of parameters, and figure 9 for the parameter set chosen here.

These results show that, in both cases, there is significant fine-tuning with respect to negative $\tilde{\mu}$. There is no significant fine-tuning with respect to C_0, and for positive $\tilde{\mu}$ the dominant fine-tuning is that with respect to the standard supersymmetry breakings. These effects are consistent with the behaviour explained in section 9. The absolute magnitude of the fine tuning is smaller when considered with respect to the invariant combinations of parameters.

11. Summary

The 3-family beta functions have been obtained for the MSSM with the NSSB terms, in a form suitable for numerical implementation, and in an economical way making use of the reparameterisation invariance.

The mSUGRA boundary conditions have been adapted for use in the context of the NSSB reparameterisation invariance, by choosing to impose the boundary conditions in the reparameterisation where they are most nearly satisfied. These boundary conditions allow the conventional mSUGRA (or CMSSM) model to be seen as a subspace of the full parameter space including NSSB terms, and the calculation of the superspectrum can be made in the conventional way.

The superparticle spectrum and a fine-tuning measure have been calculated for non-standard SUSY breaking near mSUGRA point II, where it is found that there is no significant experimental signature indicating the new terms, except where they approach their maximum allowed values.

The conventional MSSM methods are, however, found to be inadequate in this extreme region, due to the instability of the numerical method used. A more stable numerical method has been developed which solves this problem.

The phase transition in the Higgs potential occurring for large negative C_0 at mSUGRA point II suggests that this model potentially has a rich phase structure, although this appears to be of only theoretical significance, since in this phase, the Higgs VEVs are very large.

In conclusion, we see that the numerical approach used to calculate the MSSM superspectrum can be applied to this model, but there are subtle complexities which must be dealt with. As supersymmetric models continue to be developed, given the need to calculate their properties sufficiently accurately for comparison with experiment, such subtleties must be treated carefully.

Acknowledgments

I would like to thank the members of the Cambridge Supersymmetry Working Group, particularly B.C. Allanach, the author of SOFTSUSY, and B.R. Webber, my supervisor. This work was funded by the U.K. Particle Physics and Astronomy Research Council.

A. Renormalisation group equations

The 1-loop three family β-functions for the MSSM are presented below, in the reparameterisation where $\mu = 0$, with family, colour and electroweak indices suppressed. The convention for the β-functions of ref. [16] is used, i.e.:

$$\beta_x = \frac{dx}{d(\ln m)}\,, \tag{A.1}$$

where m is the renormalisation scale.

Note that for the β-functions of the scalar masses, only the additional contribution from the non-standard terms is given. These should be added to the expressions given in [16], which is denoted as β^{MSSM}. The β^{MSSM} in SOFTSUSY include some two-loop terms. For β-functions not given below, the NSSB couplings do not contribute, and the results of [16] may be used.

$$\begin{aligned}
16\pi^2\beta_{C^u} &= 3\,\mathrm{Tr}\left(Y^dY^{d\dagger}\right)C^u + \mathrm{Tr}\left(Y^eY^{e\dagger}\right)C^u + 2C^uY^{u\dagger}Y^u + Y^uY^{u\dagger}C^u + \\
&\quad + 6Y^u\,\mathrm{Tr}\left(C^uY^{u\dagger}\right) + 3Y^dY^{d\dagger}C^u + 2C^dY^{d\dagger}Y^u - \\
&\quad - C^u\left(\frac{4}{9}g_1^2 + \frac{16}{3}g_3^2\right) - 4\tilde{\mu}Y^dY^{d\dagger}Y^u - 2\tilde{\mu}Y^u\left(g_1^2 + 3g_3^2\right), \\
16\pi^2\beta_{C^d} &= 3\,\mathrm{Tr}\left(Y^uY^{u\dagger}\right)C^d + 2C^dY^{d\dagger}Y^d + Y^dY^{d\dagger}C^d + \\
&\quad + 6Y^d\,\mathrm{Tr}\left(C^dY^{d\dagger}\right) + 3Y^uY^{u\dagger}C^d + 2C^uY^{u\dagger}Y^d + 2Y^d\,\mathrm{Tr}\left(C^eY^{e\dagger}\right) + \\
&\quad + C^d\left(\frac{2}{9}g_1^2 - \frac{16}{3}g_3^2\right) - 4\tilde{\mu}Y^uY^{u\dagger}Y^d - 2\tilde{\mu}Y^d\left(g_1^2 + 3g_3^2\right), \\
16\pi^2\beta_{C^e} &= 3\,\mathrm{Tr}\left(Y^uY^{u\dagger}\right)C^e + 2C^eY^{e\dagger}Y^e + Y^eY^{e\dagger}C^e + \\
&\quad + 2Y^e\,\mathrm{Tr}\left(C^eY^{e\dagger}\right) + 6Y^e\,\mathrm{Tr}\left(C^dY^{e\dagger}\right) - \\
&\quad - 2C^eg_1^2 - 2\tilde{\mu}Y^e\left(g_1^2 + 3g_3^2\right), \\
16\pi^2\beta_{m^2_{H_1}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{H_1}} + 6\,\mathrm{Tr}\left(C^uC^{u\dagger}\right) - \left(6g_2^2 + 2g_1^2\right)\tilde{\mu}^2, \\
16\pi^2\beta_{m^2_{H_2}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{H_2}} + 6\,\mathrm{Tr}\left(C^dC^{d\dagger}\right) + 2\,\mathrm{Tr}\left(C^eC^{e\dagger}\right) - \left(6g_2^2 + 2g_1^2\right)\tilde{\mu}^2, \\
16\pi^2\beta_{m^2_{\tilde{q}_L}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{\tilde{q}_L}} + 2\left(C^uC^{u\dagger} + C^dC^{d\dagger}\right) - 4\tilde{\mu}^2\left(Y^uY^{u\dagger} + Y^dY^{d\dagger}\right), \\
16\pi^2\beta_{m^2_{\tilde{l}_L}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{\tilde{l}_L}} + 2C^eC^{e\dagger} - 4\tilde{\mu}^2Y^eY^{e\dagger}, \\
16\pi^2\beta_{m^2_{\tilde{u}_R}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{\tilde{u}_R}} + 4C^{u\dagger}C^u - 8\tilde{\mu}^2Y^{u\dagger}Y^u, \\
16\pi^2\beta_{m^2_{\tilde{d}_R}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{\tilde{d}_R}} + 4C^{d\dagger}C^d - 8\tilde{\mu}^2Y^{d\dagger}Y^d, \\
16\pi^2\beta_{m^2_{\tilde{e}_R}} &= 16\pi^2\beta^{\mathrm{MSSM}}_{m^2_{\tilde{e}_R}} + 4C^{e\dagger}C^e - 8\tilde{\mu}^2Y^{e\dagger}Y^e, \\
16\pi^2\beta_B &= \frac{2}{\tilde{\mu}}\,\mathrm{Tr}\left(3C^{u\dagger}A_u + 3C^{d\dagger}A_d + C^{e\dagger}A_e\right) + 2g_1^2M_1 + 6g_2^2M_2, \\
16\pi^2\beta_{\tilde{\mu}} &= \tilde{\mu}\,\mathrm{Tr}\left(3Y^uY^{u\dagger} + 3Y^dY^{d\dagger} + Y^eY^{e\dagger}\right) - g_1^2 - 3g_2^2.
\end{aligned} \tag{A.2}$$

B. Numerical methods

B.1 Introduction

The numerical methods used in this paper may be formally described as follows, where for clarity we note what each general variable corresponds to in our SUSY case. Let the boundary conditions at scale $x = M_X$ be

$$f(p(x)) = 0\,. \tag{B.1}$$

This equation corresponds to the mSUGRA boundary conditions, e.g. $m_q^2 = m_0^2$ and $p(x)$ corresponds to m_0, $M_{1/2}$, etc. Let the boundary conditions at scale z ($= M_Z$ and/or M_S) be

$$g(q(z)) = 0\,. \tag{B.2}$$

This condition corresponds to the physical values of the Yukawa couplings, (h_t etc.), the gauge couplings, and the REWSB condition (which constrains μ and B). The full set of parameters for the model is $\{p(t), q(t)\}$, with $p(t)$ constrained at $t = x$ and $q(t)$ constrained at $t = z$. Let q_C be the q solving eq. (B.2) and let p_C be the p which solves eq. (B.1). The differential equations (renormalisation group equations) then relate the parameters at the two scales:

$$\begin{aligned} q(z) &= R^q(q(x), p(x))\,, \\ p(z) &= R^p(q(x), p(x))\,. \end{aligned} \tag{B.3}$$

B.2 The shooting method

For a given $q(x)$, i.e. $\mu, B, Y^f \tan\beta$, and $g_{1,2,3}$, at M_X, therefore, $g(q(z))$ can be calculated numerically through R^q. The solution of the boundary value problem therefore reduces to the numerical solution of the algebraic equation $g(R^q(q(x), p_C)) = 0$ for $q(x)$. This is the "shooting method" of [23].

The shooting method cannot, however, be used for SUSY. Firstly, we have three scales at which boundary conditions are imposed: M_Z, M_S and M_X. Secondly, the boundary conditions at M_Z and M_S cannot be conveniently expressed in the form of eq. (B.2).

B.3 The drift method

In most implementations of the shooting method for SUSY, an alternative technique, which can be called the "drift method", is used: Start with a guess, q^g, for $q(x)$, i.e. $\mu(M_X)$ etc., and determine $p(x)$ from eq. (B.1). Determine $q(z)$ and $p(z)$ from eq. (B.3), the renormalisation group equations. Impose eq. (B.2) to re-determine $q(z)$. (Set the Yukawa matrices and gauge couplings from experimental data and set μ and B from the REWSB constraints.) Use the inversion of B.3, i.e. run back up to high scale, to find new values for $q(x)$ and $p(x)$, $q'(x)$ and $p'(x)$, respectively.

This corresponds to a recurrence relation $q'(x) = Q(q^g)$. Alternatively, the same procedure can be applied with a guess p^g for $p(z)$, i.e., the soft breaking parameters at M_Z, and a recurrence relation for $p(z)$.

The required values for p, q consistent with both sets of boundary conditions will be a fixed point of Q. We can hopefully determine this fixed point by making a guess for $q(x)$ and iterating the procedure until it is convergent, i.e. hoping the recurrence relation will drift to the fixed point we want. Convergence is tested for by requiring q and p not to change over a cycle. However, note that the fixed point corresponding to the physical solution is not guaranteed to be the only fixed point, nor is it guaranteed to be stable.

B.4 The single ended shooting method

When the required fixed point of Q becomes unstable, and/or the drift method drifts into a different fixed point of Q, a different approach can be used. The equation:

$$Q(q^g) - q^g = 0 \tag{B.4}$$

can be solved for q^g. In this work, the Newton-Raphson method was used to solve this equation. This has improved stability, but is not perfect, because any unrequired fixed points of Q will also correspond to roots of eq. (B.4). The method can be slow, but, if an approximate solution only is desired, one can just take initial guesses for the less important parts of q^g such as Y^e, rather than solving for them, making the method faster by reducing the parameter space to be solved.

The procedure used to generate q' also generates $p' = P(q^g)$. For q^g consistent with both sets of boundary conditions (and therefore a solution of eq. (B.4)), $p' = p_C$, and therefore $f(p') = 0$. This provides a check that one has found the desired root.

One might imagine that the boundary value problem can be reduced to the numerical solution of the algebraic equation $P(q^g) - p_C = 0$, but this turns out to be unstable: formally, it is overconstrained (there being more elements in p than in q for SUSY), but many of the constraint equations are approximately degenerate.

References

[1] S.P. Martin, *A supersymmetry primer*, `hep-ph/9709356`.

[2] H.E. Haber, *Introductory low-energy supersymmetry*, `hep-ph/9306207`.

[3] L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939.

[4] J. Rosiek, *Complete set of Feynman rules for the minimal supersymmetric extension of the standard model*, *Phys. Rev.* **D 41** (1990) 3464, erratum `hep-ph/9511250`.

[5] F.M. Borzumati, G.R. Farrar, N. Polonsky and S. Thomas, *Precision measurements at the Higgs resonance: a probe of radiative fermion masses*, `hep-ph/9712428`.

[6] F. Borzumati, G.R. Farrar, N. Polonsky and S. Thomas, *Soft Yukawa couplings in supersymmetric theories*, *Nucl. Phys.* **B 555** (1999) 53 [`hep-ph/9902443`].

[7] I. Jack and D.R.T. Jones, *Non-standard soft supersymmetry breaking*, *Phys. Lett.* **B 457** (1999) 101 [`hep-ph/9903365`].

[8] I. Jack and D.R.T. Jones, *Quasi-infra-red fixed points and renormalization group invariant trajectories for non-holomorphic soft supersymmetry breaking*, *Phys. Rev.* **D 61** (2000) 095002 [`hep-ph/9909570`].

[9] J.L. Diaz-Cruz, *Fcnc from non-standard soft-breaking terms in the MSSM*, `hep-ph/9906330`.

[10] S.P. Martin, *Dimensionless supersymmetry breaking couplings, flat directions and the origin of intermediate mass scales*, *Phys. Rev.* **D 61** (2000) 035004 [`hep-ph/9907550`].

[11] J.M. Frere, M.V. Libanov and S.V. Troitsky, *Neutrino masses from nonstandard supersymmetry breaking terms*, *Phys. Lett.* **B 479** (2000) 343 [`hep-ph/9912204`].

[12] J.M. Frere, M.V. Libanov and S.V. Troitsky, *Phenomenological aspects of nonstandard supersymmetry breaking terms*, `hep-ph/0006149`.

[13] E. Ma and M. Raidal, *Relaxing $B \to s\gamma$ constraints on the supersymmetric particle mass spectrum*, *Phys. Lett.* **B 491** (2000) 297 [`hep-ph/0006253`].

[14] F.E. Paige, S.D. Protopescu, H. Baer and X. Tata, *ISAJET 7.40: a Monte Carlo event generator for pp, $\bar{p}p$ and e^+e^- reactions*, `hep-ph/9810440`.

[15] B.C. Allanach, *SOFTSUSY: a C^{++} program for calculating supersymmetric spectra*, `hep-ph/0104145`.

[16] V. Barger, M.S. Berger and P. Ohmann, *The supersymmetric particle spectrum*, *Phys. Rev.* **D 49** (1994) 4908 [`hep-ph/9311269`].

[17] G. Gamberini, G. Ridolfi and F. Zwirner, *On radiative gauge symmetry breaking in the minimal supersymmetric model*, *Nucl. Phys.* **B 331** (1990) 331.

[18] P.M. Ferreira, *Minimisation of a one-loop charge breaking effective potential*, *Phys. Lett.* **B 512** (2001) 379 [`hep-ph/0102141`].

[19] ATLAS Collaboration, *Detector and physics performance TDR*, volume II, Technical Report CERN/LHCC 99-15, 1999 CERN.

[20] G. Corcella et al., *HERWIG 6: an event generator for hadron emission reactions with interfering gluons (including supersymmetric processes)*, *J. High Energy Phys.* **01** (2001) 010 [`hep-ph/0011363`].

[21] B.C. Allanach, J.P.J. Hetherington, M.A. Parker and B.R. Webber, *Naturalness reach of the large hadron collider in minimal supergravity*, *J. High Energy Phys.* **08** (2000) 017 [`hep-ph/0005186`].

[22] J.L. Feng, K.T. Matchev and T. Moroi, *Focus points and naturalness in supersymmetry*, *Phys. Rev.* **D 61** (2000) 075005 [`hep-ph/9909334`].

[23] W.H. Press et al., *Numerical recipes in C*, 2nd ed. C.U.P. 1992.